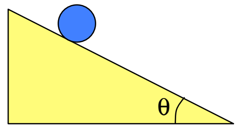
Problem 10.61

A can rolls down a 3.00 meter, 25° incline plane in 1.50 seconds.

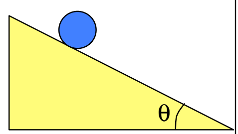


a.) Assuming *mechanical energy* is conserved, determine the can's *moment of inertia*.

NOTE: If you read the extra material at the end of the last problem, you will know that problems like this can be done by tracking the body's *center of mass*, or they can be done by assuming a *pure rotation* about the instantaneously fixed contact point. As I pointed out at the end of that section, the best approach is the one that mimics what the system is actually doing. In this case, the *center of mass* is moving and there is clearly angular acceleration *around* the center of mass. In other words, the *center of mass approach* is the reasonable one to use here.

To begin with, we need to know the *translational* and *rotational speed* of the can at the bottom of the incline. Kinematics does nicely here.

We know the *center of mass* starts from rest and makes it down the 3.00 meter ramp in 1.50 seconds. With that, we can write:



$$y_2 - y_1 = \cancelto{0}{v_1}(\Delta t) + \frac{1}{2}a_{cm}(\Delta t)^2$$

$$\Rightarrow \quad \Delta y = \frac{1}{2}a_{cm}(\Delta t)^2$$

$$\Rightarrow \quad (3.00\text{ m}) = \frac{1}{2}a_{cm}(1.50\text{ m})^2$$

$$\Rightarrow \quad a_{cm} = 2.67\text{ m/s}^2$$

and

$$v_{2,cm} = \cancelto{0}{v_{1,cm}} + a_{cm}(\Delta t)$$

$$\Rightarrow \quad v_{2,cm} = (2.67\text{ m/s}^2)(1.50\text{ s})$$
$$= 4.01\text{ m/s}$$

and

$$\omega_2 = \frac{v_{2,cm}}{R} = \frac{(4.01\text{ m/s})}{(.0319\text{ m})} = 126\text{ rad/s}$$

So *Conservation of Energy* yields:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mgy_1 + 0 = \left(\frac{1}{2}m(v_{2,cm})^2 + \frac{1}{2}I(\omega_{2,cm})^2\right) + 0$$

$$\Rightarrow \quad mg(d\sin\theta) = \left(\frac{1}{2}m(v_{2,cm})^2 + \frac{1}{2}I_{cm}\left(\frac{(v_{2,cm})}{R}\right)^2\right)$$

$$\Rightarrow \quad I_{cm} = \frac{2R^2m}{(v_{2,cm})^2}\left[g(d\sin\theta) - \frac{1}{2}(v_{2,cm})^2\right]$$

$$\Rightarrow \quad I_{cm} = \frac{2(.0319\text{ m})^2(.215\text{ kg})}{(4.01\text{ m/s})^2}\left[(9.80\text{ m/s}^2)((3.00\text{ m})\sin 25^\circ) - \frac{1}{2}(4.01\text{ m/s})^2\right]$$
$$= 1.20\text{x}10^{-4}\text{ kg}\bullet\text{m}^2$$

b.) What variable was not needed?

In looking at the derived expression, the height of the can was not used. This shouldn't be surprising. The expression for a homogeneous cylinder is $\frac{1}{2}mR^2$, which doesn't have a height term in it either. On a more conceptual level, the *moment of inertia* is associated with the way the mass is distributed about the axis of interest. If that axis is the central axis of the cylinder, then the *moment of inertia* will have to do with how the mass is positioned as you move out from that central axis, and the length will have nothing to do with anything.

c.) Why can't the expression $\frac{1}{2}mR^2$ be used here?

The can's shell is metal. The soup is, well, soup. As the overall mass of the can system is not uniformly distributed as you proceed out from the central axis, that *moment of inertia* expression doesn't hold.